International Journal of Behavioral Development

http://jbd.sagepub.com

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International Journal of Behavioral Development 2007; 31; 340

DOI: 10.1177/0165025407077755

International Journal of Behavioral Development
2007, 31 (4), 340–346
http://www.sagepublications.com

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Nonlinear latent curve models for multivariate longitudinal data

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Latent curve models have become a useful approach to analyzing longitudinal data, due in part to their allowance of and emphasis on individual differences in features that describe change. Common applications of latent curve models in developmental studies rely on polynomial functions, such as linear or quadratic functions. Although useful for describing linear forms of change and some that are nonlinear, latent curve models based on polynomial functions are not suitable for describing many developmental processes that change in a nonlinear manner. This article considers nonlinear latent curve models that permit researchers to consider a variety of nonlinear functions to characterize developmental processes. An example is provided that considers simultaneous development of two behaviors.

Keywords: development; latent curve model; longitudinal data

Latent curve models provide a flexible approach to the analysis of longitudinal developmental data. Under a latent curve model, an individual's response may be considered a function of time as an aid to understanding the development of a given behavior, or how specific features of change in a behavior may be related to covariates. When the development of a behavior follows a linear course, for instance, a model may be defined by one feature that captures the response level at a particular point in time and a second that relates to a linear change rate. More generally, responses across individuals are assumed to be dependent on a common form of change, such as one that is linear, but individuals may vary in their dependence on this common form (Meredith & Tisak, 1984, 1990). This quality of the model allows individuals to differ in their developmental trajectories. For instance, curves may vary in their response level and change rates. Depending on the statistical software used to estimate the model, individuals may be measured at unique time points, such as when individuals are different ages at each wave of measurement and the behavior is to be studied as a function of age rather than arbitrary times of measurement (Blozis & Cudeck, 1999; Mehta & West, 2000). Missing data may also be handled (see Schafer & Graham, 2002, for a review concerning missing data techniques for linear structural equation models).

Perhaps the most common application of a latent curve model is one based on a polynomial function, such as a linear or quadratic function, or a spline function linking two or more polynomial functions (see, e.g., Raudenbush & Bryk, 2002, Chapter 6). Polynomial functions, however, are not always well suited to capturing all forms of nonlinear change. Reading ability in children, for example, often changes at a nonlinear rate with a tendency for performance to stabilize at older ages (Francis, Shaywitz, Stuebing, Shaywitz, & Fletcher, 1996). An application of a quadratic function to such data might do well in capturing the increase in ability during childhood but would then imply that ability decreases thereafter. Thus, not only might a quadratic function provide a poor fit to these measures, characteristics of the function would not match theoretical considerations. A special case of a latent curve model that allows for nonlinear change in longitudinal responses is a latent basis curve model (McArdle, 1988; Meredith & Tisak, 1990). In this model, the shape of the common response form is unknown and estimated using sample data. Although some missing data may be handled, a latent basis curve model generally requires individuals be measured according to the same data collection scheme, such as when individuals are the same ages at each occasion. Practically speaking, many developmental investigations rely on data that are collected when individuals are different ages at each occasion or that have unequal intervals between assessments. Thus, the applicability of the latent basis curve model would be limited in this domain as ignoring such details of the pattern of data collection may obscure true trends in the behavioral course.

In Meredith and Tisak's (1984, 1990) formulation of the latent curve model, the function that characterizes a longitudinal response at the individual level may be a nonlinear function, such as an exponential function. There is no restriction on the chosen function with the exception that any nonlinear parameter must be fixed across individuals. Other parameters may vary between individuals. For example, for a response that is described by a power function, such as $\beta_0{}^{\beta_1}$, where β_0 denotes the response level when time is equal to 0 and β_1 is the nonlinear change rate, individuals may differ with regard to the fixed response level given by β_0 but not the nonlinear change rate, β_1. Still, the model represents a very flexible approach in that many different functions may be used to describe individual trajectories. Blozis and Cudeck (1999) proposed a related model referred to as a conditionally linear mixed-effects model that also requires that nonlinear parameters are fixed across individuals. Unlike the latent curve model proposed by Meredith and Tisak, which allows for some missing data and multiple patterns of observation, such as

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those due to observing multiple cohorts, a conditionally linear mixed-effects model allows individuals to be observed at completely unique time points, such as for cases when individuals are different ages at each measurement occasion or the intervals between assessments differ among individuals. The model proposed by Blozis and Cudeck also included a factor analysis model for a separate set of covariates that do not vary across time but together serve as indicators of a latent variable, such as self-esteem, which may be related to the characteristics defining change in the individual-level response.

In a latent curve model based on a polynomial function, a set of basis curves common to all is based on fixed and often known values. For example, when change in a behavior is linear, the first basis curve corresponds to the response level and so is set equal to unity. The second basis curve corresponds to change in the response and so is often equal to the times of measurement. An individual's response is then assumed to be a linear combination of the basis curves and a set of weights specific to the individual. The weights, representing specific change characteristics, allow individuals to vary in their response trajectories, such as having unique response levels and linear change rates. In a nonlinear latent curve model, the basis curves may be nonlinear functions of time. Similar to a model based on a polynomial, an individual's response is assumed to be a weighted linear combination of a set of common basis curves and a set of weights unique to the individual. The weights, again, represent specific features of change and so allow for individual differences in the response trajectories.

Browne (1993; also see Browne & Du Toit, 1999) proposed a structured latent curve model that, unlike the nonlinear latent curve described above, allows individuals to differ with regard to nonlinear parameters, such as allowing individuals to vary in terms of a nonlinear change rate. Missing data and unique data collection patterns across individuals are also possible (Blozis, 2004). The structured curve model is limited, however, with regard to the kind of functions that may be specified. For example, Browne (1993) describes three nonlinear functions, an exponential, logistic and Gompertz function, that may be specified under this framework. Individual-level responses are assumed to depend on features of a common curve but may do so at varying levels. Although both the latent curve model and the structured latent curve model allow for nonlinear patterns of change in a longitudinal response, the two models differ in that under a latent curve model all individual-level trajectories are assumed to follow the same form of change, whereas under a structured latent curve model only the mean response is assumed to follow a particular function.

A natural extension of a latent curve model is one that considers multiple longitudinal responses simultaneously. For example, a longitudinal response may be studied along with other measures treated as time-varying covariates. That is, change in a response, adjusted for the effects of one or more other measures that have also been measured over time, may be useful in characterizing change in a variable after accounting for variation in the response that is due to other longitudinal variables. Alternatively, a set of longitudinal responses measured at each occasion may serve as indicators of a latent measure to then be studied as a function of time. Such models, referred to as second-order latent curve models, assume that a latent variable accounts for the patterns of correlations among a set of variables. The latent measure is then studied as a function of time, similar to that done for an observed variable in a latent curve model (Chan, 1998; Duncan & Duncan, 1996; Sayer & Cumsille, 2001).

An additional approach, and one that is considered in great detail here, is one that considers multiple longitudinal processes to study how characteristics of change in one variable are related to features that describe change in another variable. For example, in studies that utilize multiple informants, such as having a parent and teacher provide developmental information about a child, longitudinal patterns may be as assessed by multiple observers to study the degree of correspondence (or lack thereof) in reports between observers. Alternatively, it may also be possible to consider regression analyses in which the random coefficients corresponding to one variable are regressed on those of another. Unlike the first two methods, the times of measurement may vary both within and between the highest sampling unit (e.g., parent and teacher dyads). The structured latent curve model has been considered for the simultaneous consideration of multiple longitudinal variables, both in a model that allows for the study of correlations between change features in one variable and change features of another (Blozis, 2004) and in a second-order model that considers a set of variables as indicators of a latent variable that may then follow a nonlinear function of time (Blozis, 2006).

This article considers nonlinear latent curve models for the study of longitudinal developmental data. Motivated by the simultaneous consideration of antisocial behaviors and academic achievement measures for a sample of adolescents, we begin by reviewing latent curve models based on polynomial functions before describing nonlinear latent curve models, including the structured latent curve model. A general approach for the simultaneous consideration of multiple longitudinal measures is then described to study how characteristics of change in antisocial behaviors are related to change in an indicator of academic performance.

Longitudinal study of antisocial behaviors and academic performance

Research suggests that antisocial behaviors during childhood and adolescence can have negative consequences for individuals and their adjustment. Thus, it is important to understand factors associated with the onset and change in antisocial behavior. Patterson, Reid, and Dishion (1992) suggest that antisocial behavior emerges out of a developmental process that is predicted by behavioral interactions with parents, siblings, and peers. Early-onset antisocial behavior appears to be particularly problematic and has been linked with difficulties at home, poor academic performance, impaired social relations, substance abuse, and other problem behaviors (Kazdin, 1987; Moffitt, 1990; Patterson, 1982; Reid, 1993; Simons, Whitbeck, Conger, & Conger, 1991). Current research suggests that one of the ways that deviant peers may have an effect is by reducing the amount of time spent on homework and other school-related tasks, which may lead to delayed or underdeveloped academic skills, such as reading ability (Kazdin, 1993; Patterson et al., 1992). Thus, we see two distinct behavior problems, antisocial behavior and reading disability, which will likely persist without effective intervention and may even exacerbate each other (Dishion & Kavanagh, 2003; Francis et al., 1996). This research suggests that it is important to examine the association between changes in reading ability (serving here as a proxy for academic achievement) and antisocial behavior to better understand their development and potential for treatment.

We consider data from the National Longitudinal Survey of Youth (NLSY) of Labor Market Experience in Youth supported by the US Department of Labor for a subset of 547 children included in the first of several cohorts participating in the study. We present measures of antisocial behavior and reading performance for children who were between 6 and 8 years old at the first assessment (mean age was 6.9 years with a standard deviation of 0.58) with planned assessments approximately every other year over an 8-year period. Slightly fewer than half of the sample (268 cases) had complete data on both measures at each of the four occasions. Antisocial behavior was measured by a subtest from the Behavior Problems Index by Zill and Peterson (Baker, Keck, Mott, & Quinlan, 1995). Scores were based on the mothers' ratings on six items concerning antisocial behaviors over the previous 3 months. Scores represent sums across items and have a potential range of 0–12 points. At the first assessment, scores ranged from 0 to 9 with a mean of 1.9. Reading performance was measured by the Peabody Individual Achievement Test (PIAT) Reading Recognition subtest, an 84-item test designed to assess word recognition and pronunciation ability. Test scores represent a sum of items correctly answered, potentially ranging from 0 to 84. Scores were divided by 10 to reduce the scale so that is would be more comparable with the measures of antisocial behaviors. At the first assessment, reading scores ranged from 0.1 to 7.2 with a mean of 2.4 ($s = 0.86$). Both measures were studied as a function of the child's age, with age measured to the nearest month with the intervals between assessments possibly varying both within and between children. Following a brief review of a latent curve model for a single measure, we then apply a multivariate version of the model to describe the nature of change in antisocial behavior in relation to change in reading ability.

Latent curve models

In a latent curve model, a set of responses, $\mathbf{y}_i = (y_{1i}, y_{2i}, \ldots, y_{ni})'$, for an individual is considered across n_i occasions, where n_i is the total number of times an individual is observed. Assuming that all individual trajectories essentially follow the same form of change, a matrix of basis functions, $\mathbf{\Lambda}_i$, is formulated based on features of the common change form. For example, if a response is assumed to change in a linear manner, the first basis function will correspond to the response level and the second to the linear change rate. The matrix is then weighted by a set of coefficients, $\boldsymbol{\eta}_i$, that represent specific change characteristics that vary between individuals. A latent curve model is given by this weighted combination of the basis functions plus a set of measurement errors specific to the individual, $\boldsymbol{\varepsilon}_i$:

$$\mathbf{y}_i = \mathbf{\Lambda}_i\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i.$$

The matrix $\mathbf{\Lambda}_i$ includes the subscript i to indicate that the matrix may vary between individuals in terms of its elements. For example, individuals may be measured a different number of times or according to different times of measurement.

The error of the model, given by $\boldsymbol{\varepsilon}_i$, represents the discrepancy between an individual's observed score and that based on the fitted trajectory. Assuming that a particular function of time accounts for the patterns in the data, the error remaining, assumed to be normal, is also assumed to be independent with constant variance across time. In other cases, it may be reasonable to allow the errors to covary, such as by allowing some form of autocorrelation (Meredith & Tisak, 1990). Between individuals, the random weights in $\boldsymbol{\eta}_i$ are assumed to be independent and normally distributed with means, denoted here as $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_k)'$ that represent the fixed-effects for the population, such as the population level and change rate, and a variance–covariance matrix Ψ. Although not required, the number of random weights is often equal to the number of fixed effects. The two are unequal, for example, when a fixed effect does not have a corresponding random coefficient, such as when individuals vary in terms of the response level but not the rate of change. The variance–covariance matrix of the random weights provides information about the extent to which individuals vary in the change characteristics and the patterns of the covariances between them. Assuming the time-specific errors and the random weights are independent, the response is assumed to have a mean and covariance structure

$$\boldsymbol{\mu}_i = \mathbf{\Lambda}_i\boldsymbol{\alpha}$$

and

$$\mathbf{\Sigma}_i = \mathbf{\Lambda}_i\mathbf{\Psi}\mathbf{\Lambda}'_i + \mathbf{\Theta}_i,$$

respectively.

Common formulations of a latent curve model: Linear and quadratic growth

A common form of the latent curve model is one based on linear change. In considering the longitudinal measure of antisocial behaviors, for example, a linear growth model with a random intercept and slope at the individual level, with the response considered as a function of the child's age, Age_i, may be specified as

$$y_{ti} = \eta_{0i} + \eta_{1i}\text{Age}_{ti} + \varepsilon_{ti}, \quad (1)$$

where, for individual i, η_{0i} represents the expected response when $\text{Age}_{ti} = 0$, η_{1i} represents the expected annual change rate for the individual, and Age_{ti} is the individual's age at time t. The individual-level coefficients, η_{0i} and η_{1i}, are assumed to be the sums of fixed and random effects: $\eta_{0i} = \alpha_0 + b_{0i}$ and $\eta_{1i} = \alpha_1 + b_{1i}$, respectively. The coefficients α_0 and α_1 denote the population response level when $\text{Age}_t = 0$ and the annual change rate, respectively. The corresponding set of random effects, b_{0i} and b_{1i}, represent the deviations of an individual's coefficients from the respective population effects. For example, an individual whose response level at $\text{Age}_t = 0$ is higher than the population will have a positive value for the random effect b_{0i}. The set of random coefficients, η_{0i} and η_{1i}, is assumed to be independent and normally distributed as

$$\begin{bmatrix} \eta_{0i} \\ \eta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} \alpha_{0i} \\ \alpha_{1i} \end{bmatrix}, \begin{bmatrix} \phi_{00} & \\ \phi_{10} & \phi_{11} \end{bmatrix}\right).$$

The variances of the random intercept, η_{0i}, and slope, η_{1i}, are denoted by ϕ_{00} and ϕ_{11}, respectively, and their covariance by ϕ_{10}. The variances of the random coefficients provide measures of the extent to which individuals vary in each change feature. The covariance between the coefficients represents the linear relationship between the response level and the linear change rate.

A second form of change often considered is a quadratic growth model that allows for nonlinear change in a response. The model is often specified to include a linear change rate and an acceleration rate. Assuming individuals vary with regard to all model coefficients, a quadratic growth model may be specified as

$$y_{ti} = \eta_{0i} + \eta_{1i}\mathrm{Age}_{ti} + \eta_{2i}\mathrm{Age}^2_{ti} + \varepsilon_{ti}, \tag{2}$$

where, for individual i, η_{0i} and η_{1i} are the expected response level and instantaneous change rate when $\mathrm{Age}_t = 0$, and η_{2i} is the acceleration rate. Between individuals, the random intercept and the random linear and quadratic time effects are assumed to be normally distributed as

$$\begin{bmatrix} \eta_{0i} \\ \eta_{1i} \\ \eta_{2i} \end{bmatrix} \sim N\left(\begin{bmatrix} \alpha_{0i} \\ \alpha_{1i} \\ \alpha_{2i} \end{bmatrix}, \begin{bmatrix} \phi_{11} & & \\ \phi_{21} & \phi_{22} & \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \right).$$

Similar to the linear growth model, the variances of the random coefficients, ϕ_{11}, ϕ_{22}, and ϕ_{33}, characterize the degree of individual differences in each change feature. The covariances between the coefficients represent the linear relationships between change features. Specifically, ϕ_{21} is the covariance between the intercept and linear time effect, and ϕ_{31} and ϕ_{32} are the covariances between the quadratic effect and the intercept and linear time effect, respectively.

Nonlinear latent curve models

Although the most common application of the latent curve model described in Meredith and Tisak (1990) is one that follows a polynomial, such as a linear or quadratic, growth model, the model also allows for a variable to change according to a nonlinear function of time. For example, a response at time t may be assumed to follow a power function:

$$y_{ti} = \eta_{1i} + \eta_{2i}\, time^{\gamma} + \varepsilon_{ti}.$$

where the coefficients η_{1i} and η_{2i} include the subscript i indicating that they may vary between individuals but the coefficient γ that appears in the exponent is assumed to be fixed across individuals. Assumptions about the errors and the random weights are analogous to that for the latent curve models based on polynomial functions. As described earlier, this form of latent curve model assumes that all individual trajectories follow the same basic form but allows individuals to vary with regard to some of the model coefficients. In contrast to this model, a structured latent curve model is based on an assumption that the mean response follows a particular function that may be nonlinear (Browne, 1993). Similar to the latent curve model, an individual's longitudinal responses are assumed to be a weighted combination of a common matrix $\boldsymbol{\Lambda}$ and a set of random weights, $\boldsymbol{\eta}_i$, that vary between individuals, plus measurement error.

Browne (1993), for example, describes a three-parameter exponential function for the mean of a longitudinal response. Assuming, for example, that mean reading performance scores follow an exponential function of a child's age, where age may vary between individuals (Blozis, 2004), a model for the means may be specified as

$$\mu_t = \gamma_1 - (\gamma_1 - \gamma_2)\exp\{-\gamma_3 \mathrm{Age}_t\}, \tag{3}$$

where μ_t is the mean response at time t, γ_1 denotes the upper asymptote and so represents the potential performance level, γ_2 denotes the response level at $\mathrm{Age}_t = 0$, and γ_3 is the nonlinear change rate. The basis functions that make up the common matrix are also given in Browne (1993). For this exponential function, the matrix is given by

$$\Lambda_i = \begin{bmatrix} 1-\exp\{-\gamma_3\mathrm{Age}_{1i}\} & \exp\{-\gamma_3\mathrm{Age}_{1i}\} & \{\gamma_1-\gamma_2\}\mathrm{Age}_{1i}\exp\{-\gamma_3\mathrm{Age}_{1i}\} \\ \vdots & \vdots & \vdots \\ 1-\exp\{-\gamma_3\mathrm{Age}_{ti}\} & \exp\{-\gamma_3\mathrm{Age}_{ti}\} & \{\gamma_1-\gamma_2\}\mathrm{Age}_{ti}\exp\{-\gamma_3\mathrm{Age}_{ti}\} \end{bmatrix},$$

where each column of the matrix corresponds to a specific aspect of change in the response. Specifically, the first column relates to the potential performance level, the second to the initial performance level, and the third to the nonlinear change rate. Then, for each individual, the columns of the matrix are weighted by a set of coefficients that are unique to the individual. The weights represent the particular aspects of change in the response.

Multivariate latent curve model

A latent curve model for a single longitudinal response may be extended to handle multiple longitudinal variables (Blozis, in press; MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997). In this formulation of the model, it is the relationships between the change characteristics of different variables that are of interest. These may be studied by the correlations between random coefficients or by regressions in which one random coefficient is regressed on another, such as when a random slope is regressed on a random intercept (Stoolmiller, 1994). Whether the response follows a polynomial or nonlinear function for the latent curve model of Meredith and Tisak (1990) or a structured latent curve model (Browne, 1993), multiple longitudinal variables may be studied by simply stacking the multivariate response set and the corresponding model components. Considering the antisocial behaviors and reading performance scores together, a multivariate response set may be formed by stacking the responses as $\mathbf{y}_i = (\mathbf{y}_{1i}, \mathbf{y}_{2i})'$ where $\mathbf{y}_{1i}$ is the set of antisocial behavioral scores and $\mathbf{y}_{2i}$ is the set of reading performance scores. A multivariate model for the two measures may be specified as

$$\begin{bmatrix} y_{1i} \\ y_{2i} \end{bmatrix} = \begin{bmatrix} \Lambda_{1i} & 0 \\ 0 & \Lambda_{ki} \end{bmatrix} \begin{bmatrix} \eta_{1i} \\ \eta_{2i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{ki} \end{bmatrix}$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}_i \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i,$$

where $\boldsymbol{\Lambda}_{1i}$ and $\boldsymbol{\Lambda}_{ki}$ are the common matrices corresponding to the antisocial behavior and reading performance scores, respectively, and the coefficients $\boldsymbol{\eta}_{1i}$ and $\boldsymbol{\eta}_{2i}$ are the random coefficient sets for the two measures, respectively. The corresponding errors are given by $\boldsymbol{\varepsilon}_{1i}$ and $\boldsymbol{\varepsilon}_{ki}$, respectively.

When two or more longitudinal measures are considered simultaneously, the measures may be related either by way of the relationships between the time-specific errors or the random change characteristics. Within variables, the errors may be assumed to be independent or follow different patterns of covariance, as described earlier when a single longitudinal measure was considered. Between variables, the errors may also be assumed to be independent or follow some pattern of covariance, such as allowing the errors within occasions and between measures to covary. The relationships between the random change characteristics of different variables may be studied by examining the covariance matrix of the random weights.

Estimation

A variety of statistical software packages, such as those used to fit linear structural equation models (e.g., AMOS, EQS, Mplus, and LISREL) may be used to fit linear latent curve models. Allowance for nonlinear constraints are necessary when considering nonlinear latent curve models of the kind described here. Beginning with LISREL version 8.8 for Windows, for example, such models may be fitted using the CO commands (Jöreskog & Sörbom, 2006). Mx, a computer software package based on matrix algebra, also allows for some nonlinear constraints. Unlike the current version of LISREL, Mx allows individuals to be observed according to completely unique time points.

A multivariate latent curve model for antisocial behaviors and reading performance

Correlations between characteristics of change in antisocial behaviors and reading ability, each as a function of the child's age, were studied using a multivariate latent curve model. As shown in Table 1, various forms of change were considered to identify functional forms that best described each behavior over the study period. Relating back to models given previously, linear (1), quadratic (2), and exponential (3) growth models were considered. For all models considered, Age was centered to 6 years by subtracting 6 from the observed ages. Model fit was based on results from deviance tests and comparisons using the Akaike information criterion (AIC). A deviance test may be considered by taking the difference in deviance values for two nested models and evaluating the difference as a chi-square statistic with degrees of freedom equal to the difference in the number of model parameters (Raudenbush & Bryk, 2002). The AIC is a relative measure of model fit that may be used for non-nested models and penalizes models based on the number of parameters. Given two competing models, that which gives the smaller value is considered preferable. Maximum likelihood estimation of the best fitting model was carried out using Mx version 1.5 (Neale, Boker, Xie, & Maes, 2003). For the time-specific errors, we assumed that within variables, the errors were normally distributed and independent between occasions with constant variance across time. Between variables, we considered a model in which the errors within occasions and between variables covaried with constant covariance across time.

Results

Based on deviance tests and the AIC, a linear growth model best described the antisocial behaviors and an exponential function best described the reading performance measures (see Table 1). For the preferred model, estimates of the fixed model coefficients with 95% confidence intervals are given in Table 2. As shown, the antisocial behavior score at age 6 for the population was estimated to be 1.73 with an estimated 95% CI of (1.55, 1.92), while the population annual change rate in scores was estimated to be 0.055 with a 95% CI of (0.021, 0.090). Thus, on average, antisocial scores increased over time, although at a slow rate. The reading performance score at age 6 for the population was estimated to be 1.71 with a 95% CI of (1.62, 1.79). Assuming the exponential function was appropriate in characterizing performance scores, the expected potential performance level was estimated to be 8.09 with an estimated 95% CI of (7.53, 8.83). The estimated change rate was 0.137 with an estimated 95% CI of (0.115, 0.160), suggesting scores increased over time but that the rate of increase slowed as children grew older.

A deviance test may be used to assess the need for a random coefficient in a model by comparing one model in which the

Table 2
Maximum likelihood estimates (MLE) of fixed effects based on a multivariate nonlinear latent curve model

Variable	*Parameter*	*MLE*	*95% CI*
Antisocial behavior	Level at 6 years	1.73	(1.55, 1.92)
	Linear change rate	.055	(.021, .090)
Reading performance	Potential level	8.09	(7.53, 8.83)
	Level at 6 years	1.71	(1.62, 1.79)
	Nonlinear change rate	.137	(.115, .160)

Notes. 95% CI is the estimated 95% confidence interval for the parameter. Antisocial behavior scores were assumed to follow a linear growth model with a random intercept and slope; reading performance scores were assumed to depend on an exponential function with allowance for individual differences in the performance level at age 6, the potential level, and change rate.

Table 1
Indices of model fit based on a multivariate latent curve model using different forms of change

Antisocial behaviors	*Reading performance*	*–2lnL*	p	*AIC*
Linear	Linear	8730.502	17	8764.502
Linear	Quadratic	8653.362	23	8699.362
Linear	Exponential	8522.559	23	8568.559
Quadratic	Linear	8725.785	23	8771.785
Quadratic	Quadratic	8528.694	30	8588.694
Quadratic	Exponential	8519.314	30	8579.314
Exponential	Linear	8724.649	23	8770.649
Exponential	Quadratic	8528.048	30	8588.048
Exponential	Exponential	8518.766	30	8578.766

Notes. –2lnL = deviance statistic, p = # of model parameters, AIC = Akaike Information Criterion (smaller values are preferred).

variance of a given random effect is assumed to be different from zero to a second model in which the variance is assumed to be equal to zero (Raudenbush & Bryk, 2002). The variances of the five random coefficients were all considered necessary in characterizing the individual-level responses based on deviance tests.

As correlations are often easier to interpret than covariances, we discuss the correlations between the random coefficients provided in Table 3. An estimated correlation of .31 suggested a slight tendency for antisocial scores at age 6 to be associated with the annual change rate such that higher levels of antisocial scores at age 6 tended to correspond somewhat with faster change rates over the study period. For reading performance scores, the nonlinear change rate was moderately related to potential reading scores ($r = -.65$) and reading scores at age 6, ($r = .76$) indicating that lower potential reading scores tended to be related to faster rates of change as was higher reading levels at age 6. Potential reading performance and performance level at age 6 were weakly correlated ($r = -.22$), suggesting a slight tendency for higher performance levels at age 6 to be related to lower potential performance levels.

Between variables, weak to moderate patterns of association were evident. Antisocial behavioral level at age 6 was negatively correlated with potential reading performance ($r = -.31$), suggesting higher levels of antisocial behaviors at age 6 were related to lower potential reading performance levels. Similarly, at age 6, antisocial behavior was negatively correlated with reading performance ($r = -.52$), suggesting a tendency for higher levels of antisocial behaviors to be related to lower performance levels at this age. Antisocial behavior at age 6 and the potential reading performance level were negatively correlated ($r = -.43$), suggesting a tendency for those with lower behavioral problems at age 6 to later have higher reading performance levels. The linear change rate in antisocial behaviors was negatively correlated with both reading performance level at age 6 ($r = -.12$) and potential level ($r = -.34$), suggesting a tendency for children with lower antisocial behavior to also have higher reading performance.

Discussion

With their ability to characterize responses at both the population and individual levels, latent curve models have become an appealing strategy for the analysis of longitudinal developmental data. A popular formulation of the model is one based on a polynomial function, such as a linear or quadratic growth model. Although there are many behaviors for which polynomial functions are well suited in describing patterns of change during periods of development, there are also many cases in which polynomial functions are theoretically inappropriate. This study considered longitudinal measures of antisocial behaviors and reading performance from late childhood to early adolescence. Whereas antisocial behaviors were best described by a linear function for the study period, reading performance was best described by an exponential function that assumed an increase in performance as children grew older with a change rate that gradually slowed over time. In contrast to a quadratic growth model that might also reasonably capture the behavioral trajectories over the observed period, an exponential function may be considered more reasonable in terms of theoretical considerations because reading ability is typically expected to slow as children develop their skills and reach their potential levels. Further, the exponential function considered for reading performance had the same number of parameters as a quadratic growth model and so was not more demanding in terms of its parameterization.

An increasingly popular application of the latent curve model is a multivariate version in which two or more longitudinal variables are simultaneously considered to study the joint associations between variables measured over time. The model does not require that the different variables be measured at the same times, nor does it require that the variables be observed the same number of times or have equal spacing between measurement occasions. Each variable may change according to a different function. The random coefficients that describe change in each variable at the individual level may then be studied to understand how characteristics of change in one variable may be related to similar or different features describing change in another variable.

The example provided in this article showed how two longitudinal variables, antisocial behaviors and reading performance, could be related in a multivariate latent curve model in which a linear growth model characterized changes in antisocial behaviors and a nonlinear function described changes in reading performance. Although the estimated associations were weak, the results did suggest a tendency for higher levels of antisocial behaviors to be related to lower reading performance at age 6 and when individuals reach their potential performance levels. This is consistent with the literature, which suggests a link between antisocial behavior and difficulties in academic performance. Results from this analysis do not, however, suggest that one behavior is the cause of the other, but rather that there may be meaningful ties between these or related behaviors.

The primary goal of this article was to illustrate the utility of nonlinear latent curve models for developmental investigations. Although we see the use of latent curve models based on polynomial functions to be a positive step in expanding the strategies for the analysis of longitudinal developmental data,

Table 3
Correlations among random coefficients at the individual level

	η_{10i}	η_{11i}	η_{20i}	η_{21i}	η_{22i}
Antisocial behavior level at 6 years, η_{10i}	1				
Antisocial behavior linear change rate, η_{11i}	.31	1			
Reading performance potential level, η_{20i}	–.43	–.34	1		
Reading performance at 6 years, η_{21i}	–.52	–.12	–.22	1	
Reading performance nonlinear change rate, η_{22i}	.15	.21	–.65	.76	1

Note. Age was centered at 6 years.

we also see a need for researchers to consider alternative functions that may better characterize development. In many cases, a nonlinear function may rely on the same number of parameters as a polynomial function and so may not be more complex in terms of parameterization. Furthermore, a nonlinear function may also be easier to interpret than a higher order polynomial or have characteristics that better relate to a given behavior. In the example presented here, the exponential function used to describe reading performance has a parameter that was related to performance level at a given age, but also included a parameter that was related to a potential performance level. Thus, the ability to estimate different types of change within a single model allowed us to represent the changes occurring in antisocial behavior and reading ability, an important step forward in the analysis of longitudinal developmental data.

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